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The Surprising Benefits of a Parallel Universe

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Suppose that the successful completion of a project requires performing n tasks, each of which has a probability of success p. The paper establishes under what conditions it may be profitable to engage in *parallel multi-tasking*, i.e. tackling each task by following two independent routes. It is found that for $\forall n > 1$ parallel multi-tasking is profitable for a wide range of parameters when costs are linear and is always profitable for convex costs.

INTRODUCTION

This paper is supposed to surprise and even baffle the reader. On the one hand, the paper can trace its origins back to an illustrious precursor[1] and on the other it can hardly refer to recent economic literature on the subject, and even then only in a very tangential way. It puts forward a simple case for re-organizing most kinds of economic activity, by claiming that there are significant surpluses waiting to be collected.

The central message of the paper is that a wide variety of organizations would benefit from pursuing critical tasks *in parallel*. Of course, at least ever since Nelson's seminal paper (1961) on the economics of parallel R&D, economists have been aware of the potential advantages of pursuing simultaneously more than one research path. It may be recalled that Nelson's punch-line is that: '... parallel development of alternative designs seems called for when the technical advances sought are large, when much additional information can be gained from prototype testing, and when the cost of a few prototypes is small relative to total systems' costs' (Nelson, 1961, p. 361).

This paper differs from the large literature spawned by Nelson's contribution in a number of important respects insofar as:

(i) the optimality of parallel tasking is shown to be independent of the size 'of the technical advances';
(ii) it is not assumed that attempting to solve any one problem in parallel yields any additional information on any other problem; and,
(iii) the optimality of parallel tasking does not depend on the cost of the early stage ('development') being a small fraction of the overall cost of the project.

The basic difference with models of systematic search (where the decision-maker, faced with various alternatives, each with a possible different payoff distribution, has to decide which and how many to explore) is that in this paper each alternative/project requires the overcoming of *more than one task* in order to succeed and that tackling any one task in parallel does not yield any information about the cost/probability of success of any other task. In other words, the paper does not deal with multiple sequential search (as, for example, van Cayseele, 1986; Vishwanath, 1992a,b), but addresses instead the

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rather unexplored area of the optimal organization of multi-task activities.[2]

An Introductory Example

Consider a project that costs w and yields a gross expected profit of $p\pi$ where p is the probability that the project succeeds. Normalize so that the net expected profit is zero, i.e. $\pi = w/p$. Assume next that this project can be completed by following more than one (statistically independent[3]) route. Specifically, assume that, in addition to the route yielding success with probability p and cost w, there is another route that succeeds with probability βp and that costs βw,[4] with $0 < \beta \leqslant 1$. Should this second route be pursued and would the answer depend on either β, p, or w?

Expected net profit from pursuing both routes *in parallel* is given by

$$p(1+\beta-\beta p)\pi - (1+\beta)w \tag{1}$$

i.e. using the normalization $\pi = w/p$

$$p(1+\beta-\beta p)\frac{w}{p} - (1+\beta)w$$
$$= -w\beta p < 0 \quad \forall p, \beta, w \tag{2}$$

This simple result would suggest that carrying out a given task by running two parallel routes may never be an efficient allocation. The next section will dispel this notion.

THE BENEFITS OF PARALLEL MULTI-TASKING

Very few economic activities succeed by overcoming a single obstacle. Typically a firm has to design, test, produce, market, and deliver any new product and each of these activities in turn would involve the solutions of many separate problems. Thus the example in the above section misses a key feature of economic (and other) activities, namely the fact that the success of a project depends on the *combined* successes of various stages.

An Introductory Example (Continued)

So let us modify the example in the first section and assume that in order for the project to yield a gross profit of π *two* separate stages have to be completed, each with a probability of success p and each costing w. Again, we normalize expected net profit to zero:

$$p^2\pi - 2w = 0 \tag{3}$$

As before, assume that each of the two subprojects is carried out *in parallel*, with the original resources yielding success with probability p and costing w being coupled with other resources that succeed with probability βp and that cost βw. I call this arrangement *parallel multi-tasking*. In order not to bias the analysis in favour of parallel multi-tasking, we constrain β not to exceed unity, i.e. the additional parallel route to success cannot be more efficient than under single multi-tasking. The structure diagram for parallel multi-tasking is given in Figure 1.

Expected net profit is now given by

$$p^2(1+\beta-\beta p)^2\pi - 2(1+\beta)w \tag{4}$$

i.e. considering the normalization (3),

$$E(\pi) = 2w[(1+\beta-\beta p)^2 - (1+\beta)]$$
$$= 2w\beta[(1-2p) + \beta(1-p)^2] \tag{5}$$

From (5) we can see that if $p \leqslant 0.5$ then the addition of a parallel path is *always profitable irrespective of its efficiency* (i.e. as long as $\beta > 0$). Identical replication, i.e. when $\beta = 1$ is profitable for a somewhat larger range, namely $p \leqslant 0.585$ as illustrated in Figure 2.

Is the two-obstacle case as much an anomaly as the one-obstacle case?

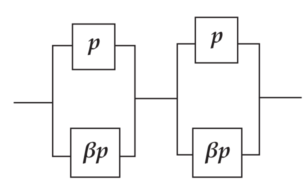

Figure 1. Parallel 2-tasking.

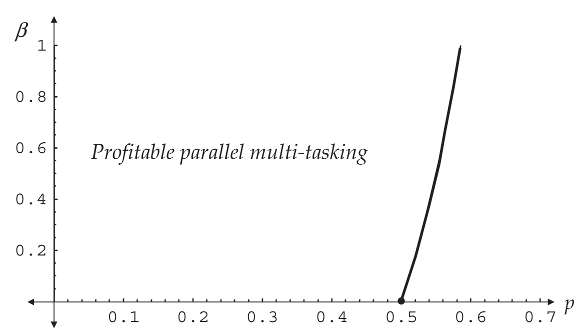

Figure 2. Profitable parallel multi-tasking (two-obstacle case).

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Parallel Multi-Tasking: The General Case

If the success of a project requires the successful completion of n tasks, then expected net profits for single and parallel multi-tasking are, respectively:

$$p^n\pi - nw \tag{6}$$

$$p^n(1+\beta-\beta p)^n\pi - n(1+\beta)w \tag{7}$$

Using the same normalization as before, (7) can be written as

$$nw[(1+\beta-\beta p)^n - (1+\beta)] = nw\beta\left[(n-1) - np + \sum_{k=2}^{n}\binom{n}{k}(1-p)^k\beta^{k-1}\right] \tag{8}$$

This shows that a *sufficient* condition for parallel multi-tasking to be profitable is that $p \leqslant (n-1)/n$. Moreover, the larger the n the larger the region in (β, p) space where parallel multi-tasking is profitable, as shown in Figure 3, where $\beta^*(p; n)$ is the locus along which parallel multi-tasking yields zero profit.

Having shown that for very significant ranges of β and p parallel multi-tasking is more profitable than single multi-tasking, we have to address the question of *how much* more profitable it may be. In fact, if it turned out that, compared with some relevant benchmark, the net benefits of parallel multi-tasking are positive but small, it could be argued that the model is not robust, insofar as the cost of running parallel activities (which we have assumed away) is accounted for, parallel multi-tasking may be profit-reducing.[5] An obvious benchmark is the total cost in the single multi-tasking case, nw, which we have normalized to be equal to expected gross profit, $p^n\pi$. In view of (8), it is easy to see that net expected profit in the parallel multi-tasking case is given by

$$nw \times g(\beta, p, n) \tag{9}$$

where

$$g(\beta, p, n) \equiv \beta\left[(n-1) - np + \sum_{k=2}^{n}\binom{n}{k}(1-p)^k\beta^{k-1}\right] \tag{10}$$

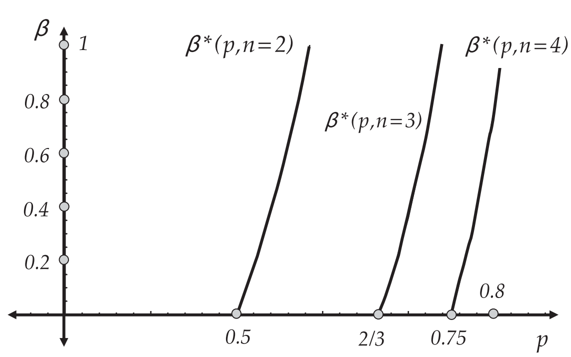

Figure 3. Profitability of parallel multi-tasking increases with n.

A few numerical examples suffice to show that parallel multi-tasking is often not only more profitable than single multi-tasking, but also by a very large amount, thereby justifying our modelling choice of ignoring the additional cost of running parallel routes to success (Table 1).

Table 1. Gain from Parallel Multi-tasking as Percentage of Total Cost under Single Multi-tasking

n	$p=2/5$	$p=1/2$	$p=2/3$
$\beta=0.5$			
2	19%	6%	—
3	69%	45%	8%
4	135%	94%	35%
$\beta=0.75$			
2	35%	14%	—
3	129%	84%	20%
4	267%	182%	69%
$\beta=1$			
2	56%	25%	—
3	209%	137%	37%
4	455%	306%	116%

Figures 4–6 plot $g(\beta, p, n)$ for three key values of β, i.e. $\beta = 0.5, 0.75, 1$.

In view of the substantial surplus generated by parallel multi-tasking when a best-practice technique is coupled with another of equal or lower efficiency, it is not surprising that parallel multi-tasking may be superior to single multi-tasking even if it employs *only* lower-efficiency units.

A simple example may suffice. Consider the two-task case and let (3) hold, i.e. single multi-tasking yields zero expected net profit. Parallel multi-tasking that uses four units with probability of success λp ($\lambda < 1$) and cost λw each (what we call *degraded replication*) is profitable provided that:

$$(2\lambda p - \lambda^2 p^2)^2\pi - 4\lambda w > 0 \tag{11}$$

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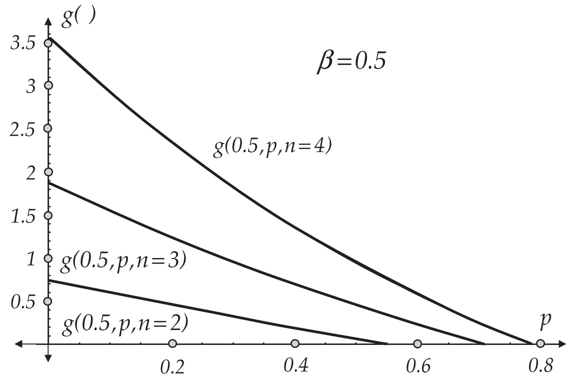

Figure 4. Net profitability of multi-tasking.

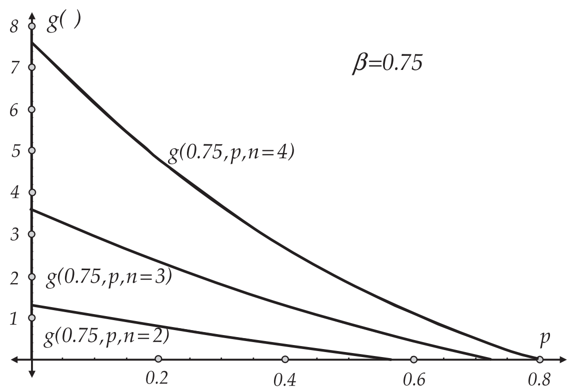

Figure 5. Net profitability of multi-tasking.

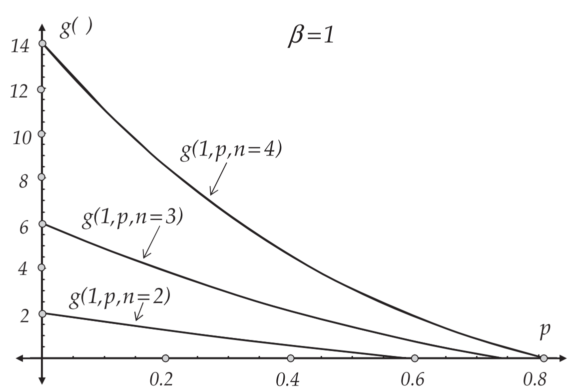

Figure 6. Net profitability of multi-tasking.

i.e. using (3):

$$2w\lambda[\lambda(2-\lambda p)^2 - 2] > 0 \tag{12}$$

Thus, degraded replication is profitable for all pairs (λ, p) that satisfy

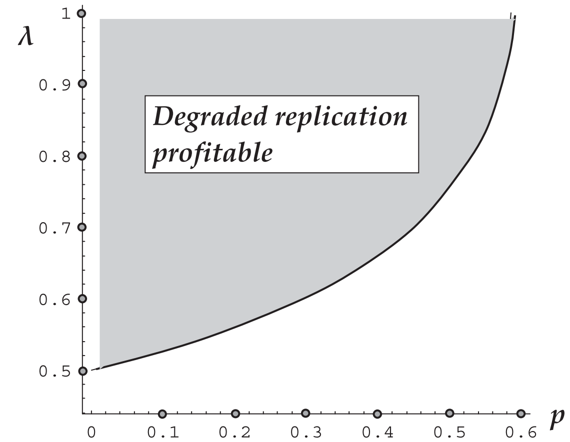

Figure 7. Profitable degraded replication (two-task case).

$$\frac{2}{\lambda}\left(1-\sqrt{\frac{1}{2\lambda}}\right) > p < 2-\sqrt{2} \tag{13}$$

i.e. for all pairs (λ, p) that fall in the shaded area in Figure 7.

The fact that deploying degraded units in parallel can be more efficient than using a series of highest-efficiency units is worth emphasizing, as it adds weight to the superiority of parallel multi-tasking. Indeed, it is a well-known result in the statistical theory of reliability that the probability of success of a system made up of a series of two-unit modules in parallel is *minimized* when all units have the same probability of success.[6] Therefore, the existence of a significant range of values of λ and p where degraded replication is more profitable than single multi-tasking is somewhat surprising insofar as we are comparing the *least* efficient parallel multi-tasking arrangement with the *most* efficient single multi-tasking organization.

OPTIMAL PARALLEL MULTI-TASKING WITH CONVEX COSTS

We can now relax the assumption of linear costs and work with a richer model that determines endogenously the 'quality' (i.e. the probability of success) of the individual units deployed to carry out a multi-stage project.

Assumption 1:

(i) The cost of performing a task i that succeeds with probability p_i is given by $c(p_i) = p_i^{\vartheta}$.

(ii) $n \geqslant 2$ distinct statistically independent tasks have to be successfully completed in order for the project to yield gross profits π.

Under single multi-tasking the profit-maximizer will solve the following program:

$$\max_p \; \{p^n \pi - np^\theta\} \tag{14}$$

For the second-order condition to hold, and for p not to exceed unity, the following two restrictions must hold:

$$\theta > n, \quad \theta > \pi \tag{15}$$

The equilibrium probability of success (for each identical unit) is given by

$$p_S^* = \left(\frac{\pi}{\theta}\right)^{1/(\theta - n)} \tag{16}$$

and the equilibrium expected net profit by

$$(\theta - n)\left(\frac{\pi}{\theta}\right)^{\theta/(\theta - n)} \tag{17}$$

In the case of parallel multi-tasking the profit-maximizer will solve the following program:

$$\max_p \; E(p) \equiv p^n (2 - p)^n \pi - 2np^\theta \tag{18}$$

$$\frac{\mathrm{d}E}{\mathrm{d}p} = 0 \Rightarrow \frac{\pi}{\theta}(2 - p)^{n-1}(1 - p) = p^{\theta - n} \tag{19}$$

Notice that in both maximization programs, we assume that it is always optimal to use identical units (i.e. $p_1 = p_2 = \cdots = p_n = p$). The rationale for this assumption is explained in the following result (which may be of independent interest):

Lemma 1:
The minimum of a Schur-convex function $f(\mathbf{p})$ is attained at $p_1 = p_2 = \cdots = p_n = \bar{p} = \frac{1}{n}\sum_{i=1}^n p_i$.

It is well known that if vector $\mathbf{p}$ majorizes vector $\mathbf{p}'$ and $f(\mathbf{p})$ is Schur-convex, then $f(\mathbf{p}) > f(\mathbf{p}')$. Obviously for any vector $\mathbf{p} \neq \overline{\mathbf{p}}$, where $\overline{\mathbf{p}} = \{\bar{p}, \ldots, \bar{p}\}$, it is the case that $\mathbf{p} \succ \overline{\mathbf{p}}$.

Lemma 2:
Under Assumption 1, expected net profit from parallel multi-tasking, $h(\mathbf{p})\pi - C(\mathbf{p})$, is maximized when the probability of success of all units is identical.

The reliability function $h(\mathbf{p})$ of a series of modules made up of two units in parallel is known to be is Schur-concave,[7] i.e. $\mathbf{p}' \succ \mathbf{p}'' \Rightarrow h(\mathbf{p}') < h(\mathbf{p}'')$. Considering that the cost function $C(\mathbf{p}) = p_1^{\vartheta} + \cdots + p_n^{\vartheta}$ is Schur-convex and therefore $-C(\mathbf{p})$ is Schur-concave, it follows that the expected profit function $h(\mathbf{p})\pi - C(\mathbf{p})$ is Schur-concave and therefore is maximized at $p_1 = \cdots = p_n$.

We can now prove the following:

Theorem 1:
If Assumption 1 holds, then expected net profits under parallel multi-tasking are always higher than under single multi-tasking.

Proof:
See appendix, where we prove Theorem 1 by showing that even though the optimal probabilities of success of a task under single and parallel multi-tasking are typically different, i.e. $p_S^* \neq p_T^*$, the optimization over probabilities is of second order compared with the optimization over organizational structure, insofar as *parallel* multi-tasking yields a higher net expected profit even if tasks are carried out at the probability of success that is optimal under *single* multi-tasking. □

To give an idea of the magnitude of the extent to which parallel multi-tasking dominates single multi-tasking, a few numerical examples may suffice (Figure 8) and Table 2.

Another, indirect, way of appreciating the intrinsic superiority of parallel multi-tasking compared with single multi-tasking is by comparing net expected profits when an additional constraint is imposed on parallel multi-tasking, namely that the cost of the project cannot exceed the optimal cost of the project under single multi-tasking, i.e.

$$\begin{aligned} C(p_S^*) = C(p_T) &\rightarrow n(p_S^*)^{\vartheta} \\ &= 2n(p_T)^{\vartheta} \rightarrow p_T = 2^{-1/\vartheta} p_S^* \end{aligned} \tag{20}$$

Given that the optimal probability of success under parallel multi-tasking is often greater than under single multi-tasking, the restriction imposed by (20) is a telling one. Nevertheless it is not difficult to show that, even under (20), parallel multi-tasking is the superior organization.

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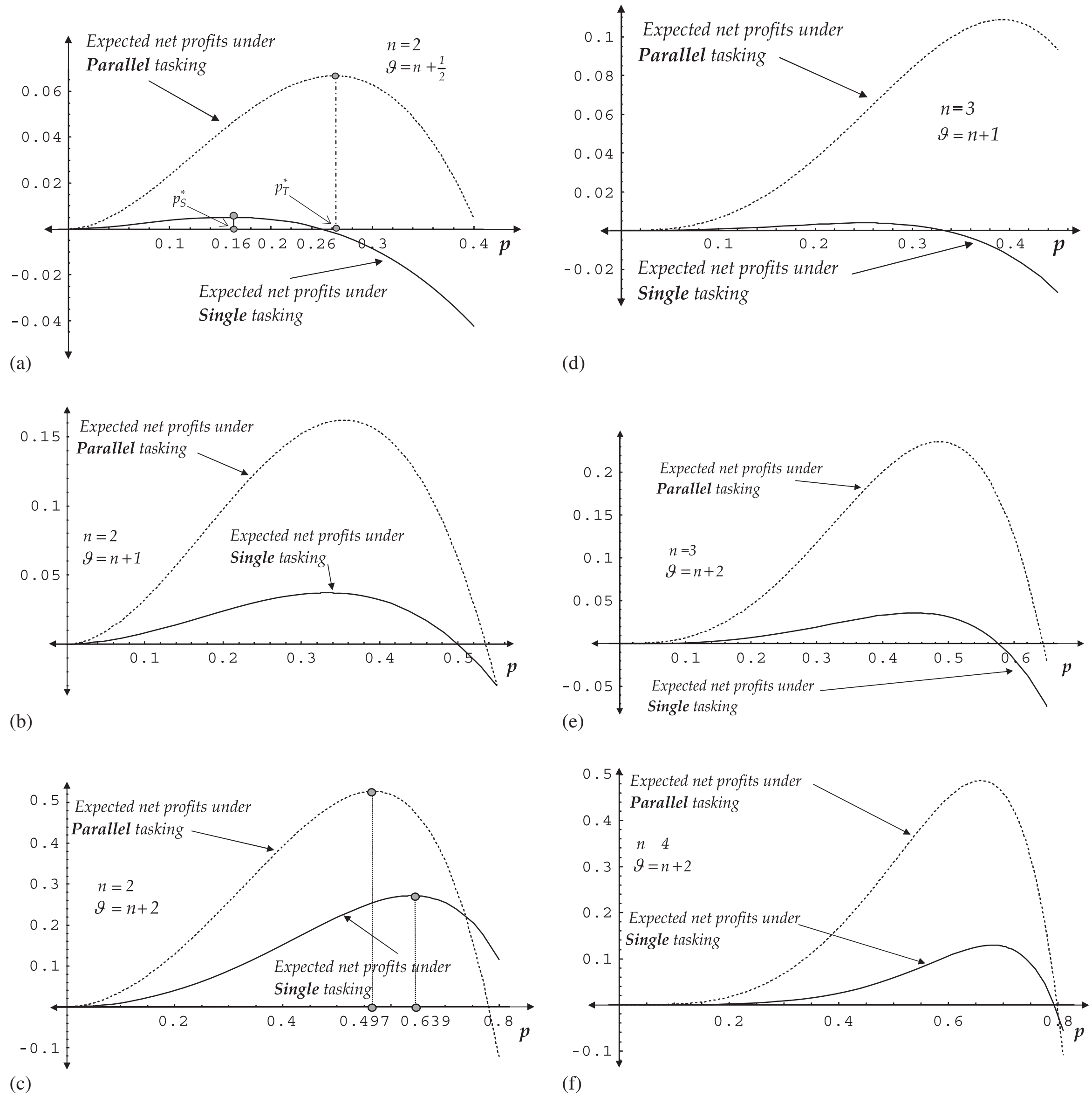

Figure 8. Profitability under single and parallel tasking.

Corollary:
Under Assumption 1, expected net profits are always higher under parallel multi-tasking than under single multi-tasking even if the total cost of the project is constrained to equal the total cost under single multi-tasking.

Proof:
Under Assumption 1 and (20), we need to show that

$$p_T^n(2-p_T)^n \geqslant p_S^{*n} \tag{21}$$

i.e. using (20), setting $\vartheta \equiv n+\varepsilon$ and simplifying,

$$G(n,\varepsilon) \equiv 2 - 2^{-(n+2\varepsilon)/\varepsilon(n+\varepsilon)} - 2^{1/(n+\varepsilon)} \geqslant 0 \tag{22}$$

Noting that (i) $\lim_{\varepsilon\to 0} G(n,\varepsilon) = 2 - 2^{1/n}$; (ii) $\lim_{n\to\infty} G(n,\varepsilon) = 1 - 2^{-1/\varepsilon}$; and (iii) $\lim_{\varepsilon\to\infty} G(n,\varepsilon) = 0$, it follows that (22) is satisfied for all $0<\varepsilon<\infty$ and all $n \geqslant 2$. □

CONCLUSIONS

By using what is probably the simplest model of multi-task organization, I hope to have shown that

Table 2. Relative Superiority of Parallel Tasking under Convex Costs

	$\frac{\tau(p_T)}{\sigma(p_S^*)}$ relative profitability of parallel compared to single multi-tasking		Overall probability of success		Optimal probability of success, p_i^*	
	$p_T = p_S^*$	$p_T = p_T^*$	Single	Parallel	Single	Parallel
$n = 2, \vartheta = 2.5$	8.928	13.048	0.025	0.207	0.16	0.262
$n = 2, \vartheta = 3$	4.333	4.375	0.111	0.339	0.333	0.354
$n = 2, \vartheta = 4$	2.5	2.578	0.25	0.497	0.5	0.457
$n = 3, \vartheta = 3.5$	37.21	529.9	0.0005	0.1467	0.081	0.312
$n = 3, \vartheta = 4$	15.43	27.83	0.015	0.2548	0.25	0.395
$n = 3, \vartheta = 5$	6.35	6.58	0.089	0.397	0.447	0.485
$n = 4, \vartheta = 4.5$	113.5	66.200	0.0000	0.131	0.049	0.369
$n = 4, \vartheta = 5$	44.48	281.8	0.001	0.213	0.2	0.434
$n = 4, \vartheta = 6$	15.23	20.82	0.027	0.342	0.408	0.515

$\tau(p_T) \equiv$ expected net profits under parallel multi-tasking when probability of success $= p_T$. $\sigma(p_S^*) \equiv$ expected net profits under single multi-tasking when probability of success $= p_S^*$. Asterisks indicate optimal choices.

for the fairly wide range of activities where success requires the overcoming of more than one obstacle it is very often the case that tackling each task in parallel yields substantial net benefits. The superior efficiency of 'parallelism' is, of course, taken for granted by both applied and pure statisticians of systems reliability. Perhaps it is surprising that economists only recently have started to pay serious attention to the issue of optimal *organization* (as opposed to optimal *allocation*) of resources, given that (i) the *process* of economic research is a perfect example of a multi-task activity that benefits greatly from parallel multi-tasking,[8] and (ii) the *funding* of economic research is a prime example of sub-optimal organization.[9]

APPENDIX

Proof of Theorem 1:

We have to establish that

$$\begin{aligned}\tau(p_S^*) &\equiv p_S^{*n}(2-p_S^*)^n\pi - 2np_S^{*\vartheta}\\ &\geqslant p_S^{*n}\pi - np_S^{*\vartheta} \equiv \sigma(p_S^*)\end{aligned} \tag{A1}$$

W.l.o.g. we can normalize $\pi = 1$ and, in view of (16), re-write (A1) as

$p_S^{*n}[(2-p_S^*)^n - 1] \geqslant np_S^{*\vartheta}$

i.e. setting $\vartheta \equiv n+\varepsilon, \varepsilon > 0$

$$(2-p_S^*)^n - 1 \geqslant np_S^{*\varepsilon} = \frac{n}{n+\varepsilon} \tag{A2}$$

Thus, we can redefine the inequality posited in Theorem 1 as:

$$F(n,\varepsilon) \equiv \left(2 - \left(\frac{1}{n+\varepsilon}\right)^{1/\varepsilon}\right)^n - 1 - \frac{n}{n+\varepsilon} \geqslant 0 \tag{A3}$$

Notice first that, as $\partial F/\partial n > 0$,[10] in order to prove (A3), it suffices to show that it holds for the smallest feasible n, i.e. $n = 2$:

$$\left(2 - \left(\frac{1}{2+\varepsilon}\right)^{1/\varepsilon}\right)^2 - 1 - \frac{2}{2+\varepsilon} \geqslant 0 \tag{A4}$$

We are going to show that (A4) holds for all $1 \geqslant p > 0$ in a series of steps.

Step one: (A4) holds for $p_S^* \leqslant \frac{1}{2}$.

To see this, notice that (A4) holds *a fortiori* if the following holds:

$$\left(\frac{3}{2}\right)^2 \geqslant 1 + \frac{2}{2+\varepsilon}, \text{ i.e.} \tag{A5}$$

$10 + 5\varepsilon \geqslant 8$, which clearly holds for $\forall\varepsilon \geqslant 0$.

Step two: (A4) holds for $p^{(i)} > p_S^* > \frac{1}{2}$.

As $p_S^* > \frac{1}{2}$, it is the case (from (16)) that $\varepsilon > 2$. Therefore (A4) holds *a fortiori* if the following holds:

$(2-p_S^*)^2 \geqslant \frac{3}{2}$, i.e. taking square roots,

$$p_S^* \leqslant 2 - \sqrt{\tfrac{3}{2}}, \text{ i.e. } p_S^* \leqslant 0.775255 \equiv p^{(i)} \tag{A6}$$

Step three: (A4) holds for $p^{(ii)} > p_S^* > p^{(i)} = 0.775255$.

In step three (and all subsequent steps), we follow the procedure in step two, namely as $p_S^* >$

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0.775255, it is the case (from (16)) that $\varepsilon > 9.64271$. Therefore, (A4) holds *a fortiori* if the following holds:

$$(2-p_S^*)^2 \geqslant 1+\frac{2}{2+9.64271}, \text{ i.e. taking square roots}$$

$$p_S^* \leqslant 2-\sqrt{1.17178}, \text{ i.e.}$$

$$p_S^* \leqslant 0.917512 \equiv p^{(\mathrm{ii})}$$

Proceeding in a similar fashion, we can show that:

Step four: $p^{(\mathrm{iii})} = 0.97878 > p_S^* > p^{(\mathrm{ii})} = 0.917512$.

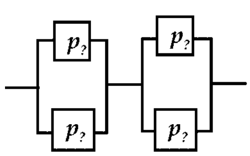
Step five: $p^{(\mathrm{iv})} = 0.996184 > p_S^* > p^{(\mathrm{iii})} = 0.97878$.

Finally, we notice that the whole process is converging in view of the fact that:

$$\lim_{\varepsilon\to\infty}\left[\left(2-\left(\frac{1}{2+\varepsilon}\right)^{1/\varepsilon}\right)^2 - 1 - \frac{2}{2+\varepsilon}\right] = 0 \qquad \text{(A8)}$$

NOTES

1. The theory underpinning this paper, the mathematical theory of system reliability, was first developed by John von Neumann (1956).
2. For a paper close to the spirit of the present one, see Moldovanu and Sela (2006).
3. Throughout the paper we maintain the assumption that the probabilities of success are i.i.d.
4. In this simple linear cost model the following simplifying assumptions are made (each of which can be dispensed with w.l.o.g.): (i) in order to avoid the trivial case $p = 1$ the zero profit-condition holds at $p < 1$; (ii) therefore the cost of resources yielding a probability of success βp is equal to βw for $0 < \beta \leqslant 1$ and ∞ otherwise.
5. This is the reason why in this paper we do not consider the more general case when each task is tackled in m ($\geqslant 2$) ways (even though even stronger results could be established). In fact, it could be argued that massive parallelism could involve substantial costs of coordination that could reverse the organizational superiority of parallel multi-tasking.
6. Let $n = 2$ (two-task case) and consider the problem of allocating four units with individual probabilities of success $\mathbf{p} = \{p_1, p_2, p_3, p_4\}$ with $p_1 > p_2 > p_3 > p_4$ to the following structure:



The probability that the whole project succeeds is given by $\phi(\mathbf{p}) = [1-(1-p_i)(1-p_j)][1-(1-p_s)(1-p_t)]$; $i \neq j \neq s \neq t$. Consider the vector $\bar{\mathbf{p}} = \{\bar{p}, \bar{p}, \bar{p}, \bar{p}\}$, where $\bar{p} = \frac{1}{4}(p_1+p_2+p_3+p_4)$. Clearly $\mathbf{p}$ majorizes $\bar{\mathbf{p}}$ (i.e. $\mathbf{p} \succ \bar{\mathbf{p}}$, see Marshall and Olkin (1979) for definitions) and $\phi(\mathbf{p})$ is strictly Schur-convex; therefore, $\phi(\mathbf{p}) > \phi(\bar{\mathbf{p}})$.

7. See, for example, El-Neweihi *et al.* (1986).
8. The analysis of research is as a 'multi-component system', see La Manna (2004).
9. Consider the following example: a funding body receives two separate funding proposals for the attainment of a given research objective with a gross payoff of £18 m. Success requires the overcoming of two equally difficult obstacles. Proposal 1 (the 'good' proposal) envisages two units each costing £1 m tackling one obstacle each with an individual probability of success equal to $\frac{1}{3}$. Thus, the net expected return is $\frac{1}{3}\times\frac{1}{3}\times £18\,\mathrm{m} - 2\times £1\,\mathrm{m} = 0$. Proposal 2 (the 'bad' proposal) pursues two statistically independent routes (independent of each other *and* of the routes pursued by proposer 1) that are half as efficient and half as costly (i.e. in the notation of the second section, $\beta = 0.5$), so that its net expected return is $\frac{1}{6}\times\frac{1}{6}\times £18\,\mathrm{m} - 2\times £0.5\,\mathrm{m} = -£0.5\,\mathrm{m}$. Most funding bodies would approve Proposal 1 and reject Proposal 2, whereas the correct choice, of course, would be to fund them both, provided they are run *in parallel* (in which case the net expected return would be $(\frac{4}{9})^2 \times £18\,\mathrm{m} - 2\times £0.5\,\mathrm{m} - 2\times £1\,\mathrm{m} = £\frac{5}{9}\,\mathrm{m}$). For details, see La Manna (2008)
10.

$$\begin{aligned}\frac{\partial F}{\partial n} \;=\;& \frac{n}{(n+\varepsilon)^2} + \left(2-\left(\frac{1}{n+\varepsilon}\right)^{1/\varepsilon}\right)^n \\ &\times\left(\frac{n\left(\frac{1}{n+\varepsilon}\right)^{(1+\varepsilon)/\varepsilon}}{\varepsilon\left(2-\left(\frac{1}{n+\varepsilon}\right)^{1/\varepsilon}\right)}\right. \\ &\left.+\left(2-\left(\frac{1}{n+\varepsilon}\right)^{1/\varepsilon}\right)\mathrm{Log}\left[2-\left(\frac{1}{n+\varepsilon}\right)^{1/\varepsilon}\right]\right. \\ &\left. -\frac{1}{n+\varepsilon} > 0\right.\end{aligned}$$

The inequality follows because the first two terms are positive, the third is positive and strictly greater than one, and the last is negative and strictly less than one.

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