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WHY ARE PROFITS CORRELATED WITH CONCENTRATION?

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In a *free-entry non-cooperative* oligopoly a correlation between profitability and concentration arises from the indivisibility of firms. Although a 'perverse' negative correlation is possible, most distributions of entry fees across industries yield a *positive* correlation.

1. At first sight there is no mystery in the commonly observed empirical correlation between concentration and profitability. Most industrial economists, if asked to provide a theoretical explanation for it, would reply that: (i) in a cooperative model, high concentration leads naturally to high profitability because of increased collusion – the fewer firms there are, the easier collusion is, thereby raising joint and individual profits, (ii) in a non-cooperative model, if concentration is defined as an inverse function of N, the number of firms in the industry, then, provided firms choose output levels simultaneously, it can be shown [see Seade (1980)] that profits per firm decline as N rises.

It is interesting to note that neither of these explanations refers explicitly to entry, and for a good reason: if free entry is assumed, then, in so far as the latter implies *zero* super-normal profits *irrespective* of the value of N, there should be no correlation at all between concentration and profitability. It follows that, according to the received wisdom, no theoretical explanation of the concentration–profitability link is available if one makes the reasonable joint assumption that there exists free entry and firms behave non-cooperatively [for an interesting model with both free entry and collusion, see Brander and Spencer (1985)].

The aim of this note is to show that, because of the integer constraint arising from the indivisibility of firms, concentration and profitability are correlated even in a non-cooperative oligopoly with free entry. More specifically, it will be shown that super-normal profits per firm depend on two parameters:

(a) the size of entry fees; the resulting 'size effect' is unambiguously positive in the sense that, ceteris paribus, industries with high entry fees are characterized by high concentration and high profits per firm;

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(b) the extent to which the integer constraint is binding; the sign of the resulting 'integer constraint effect' is ambiguous and, in some special albeit unlikely cases, may even offset the 'size effect' and produce a 'perverse' negative correlation between concentration and profitability.

Finally, it will be shown that, provided the distributions of entry fees across industries are 'well-behaved' (in a sense made precise below), then cross-industry regressions will show, on average, a positive correlation between concentration and profitability.

An informal preview of the argument may be useful at this stage.

Suppose that the number of firms sustained by a free-entry equilibrium in industry i (respectively, j) is 2.5 (4.5); clearly only two (four) firms can be active on the market, each earning super-normal profits. One would expect, ceteris paribus, the more concentrated industry i to be more profitable than industry j. This is because, figuratively speaking, the spoils of half a firm are worth more in industry i and, moreover, are shared among fewer firms. However, it is not difficult to see that, because of the integer constraint, more concentrated industries may turn out to be less profitable. Let n_i and n_j be respectively 2.01 and 3.99; in industry i profits will be close to zero, whilst the three firms active in industry j will share the non-negligible profits that would have accrued to the 'ninety-nine hundredths of a firm' which would have existed, had it not been for the integer constraint.

What follows formalizes the intuitive argument sketched above.

2. Consider an economy with M free-entry industries indexed by j ($j = 1, \ldots, M$). Assume (for simplicity only) that each industry is faced with a linear inverse demand function for its homogeneous product and that each firm has constant marginal costs, i.e.,

$$P_j = a_j - b_j \sum_j^{n_j} x_{ij}, \qquad i = 1, \ldots, n_j, \tag{1}$$

$$C_{ij}(x_i) = c_j x_{ij} + F_j, \tag{2}$$

where

x_{ij} = ith firm's output level in industry j,
n_j = number of firms in industry j, and
F_j = industry-specific fixed entry fee in industry j (on a per-period basis).

In order to focus on inter-industrial differences in entry fees, it may be assumed that all industries are identical in all respects but entry fees (nothing of substance hinges on this notation-saving assumption),

$$a_j = a, \qquad b_j = b, \qquad c_j = c \quad \forall j. \tag{3}$$

Notice that, as firms are identical, the reciprocal of the number of firms can be used as an unambiguous index of concentration. Given the number of firms in industry j, a Cournot–Nash equilibrium is established. Straightforward calculations show profits per firm to be

$$\Pi_j = (a-c)^2 / b(n_j + 1)^2 - F_j. \tag{4}$$

The free-entry equilibrium number of firms in industry j, n_j^*, is determined by the following zero-profits condition:

$$F_j = (a-c)^2/b(n_j^*+1)^2 \tag{5}$$

Of course the probability of n_j^* being an integer is a set of measure zero; thus let n_j^c be the integer-constrained number of firms in industry j, i.e., the largest integer not exceeding n_j^*.

It can be seen that industrial structure is modelled as a perfect equilibrium of a two-stage game: in the first stage firms simultaneously decide whether to incur the non-recoverable fixed entry fee F_j. In the second stage, those firms (numbering n_j^c) that have paid F_j choose their profit-maximizing output levels.

Actual profits per firm, Π_j^c, are given by

$$\Pi_j^c = (a-c)^2/b(n_j^c+1)^2 - (a-c)^2/b(n_j^*+1)^2, \tag{6}$$

i.e., defining $N_j \equiv n_j^c + 1$ and $n_j^* - n_j^c \equiv \mu_j \in (0, 1)$, we obtain

$$\Pi_j^c = (a-c)^2(2N_j+\mu_j)\mu_j/bN_j^2\left[N_j^2 + (2N_j+\mu_j)\mu_j\right]. \tag{7}$$

It can be easily verified that, for any $\mu_i \leqq \mu_k$,

$$\Pi_i^c \gtreqless \Pi_k^c \Leftrightarrow n_i^c \lesseqgtr n_k^c. \tag{8}$$

According to (8), concentration – as measured by $1/n_j^c$ – and profitability – as measured by Π_j^c – are *positively* correlated.

However, from (7) it can also be seen that actual profits depend not only on the size of fixed entry fees ('*size effect*'), but also on the difference between the free-entry equilibrium of firms with and without fractional entry, i.e., on μ_j ('*integer constraint effect*').

Whilst the size effect always leads to a positive correlation between concentration and profits [see (8)], the integer constraint effect can work in the opposite direction; indeed, cases may arise in which the latter more than offsets the former, giving rise to an overall *negative* correlation between concentration and profitability. [1]

In order to show that, on average, the size effect can be expected to be stronger than the integer constraint effect, thereby accounting for the observed positive correlation between profits and concentration, the following facts should be taken into account:

Fact 1. Industry j will sustain n_j^c firms at a free-entry equilibrium iff

$$F_j \in \left[(a-c)^2/b(n_j^c+2)^2, (a-c)^2/b(n_j^c+1)^2\right) \equiv \left[F_j^{\min}, F_j^{\max}\right).$$

Fact 2. The length of the above half-open interval falls with n_j^c.

Fact 3. For any given n_j^c, Π_j^c is an increasing concave function of μ_j [as shown in fig. 1(a)].

[1] Consider the following example: $a = 2$, $b = c = 1$, $F_1 = 2.1^{-2}$, $F_2 = 3.9^{-2}$. From (5) it follows that $n_1^c = 1$, $n_1^c = 2$, $\mu_1 = 0.1$, $\mu_2 = 0.9$. Using (6) we can compute actual (super-normal) profits: $\Pi_1^c \simeq 0.0232 < \Pi_2^c \simeq 0.0456$.

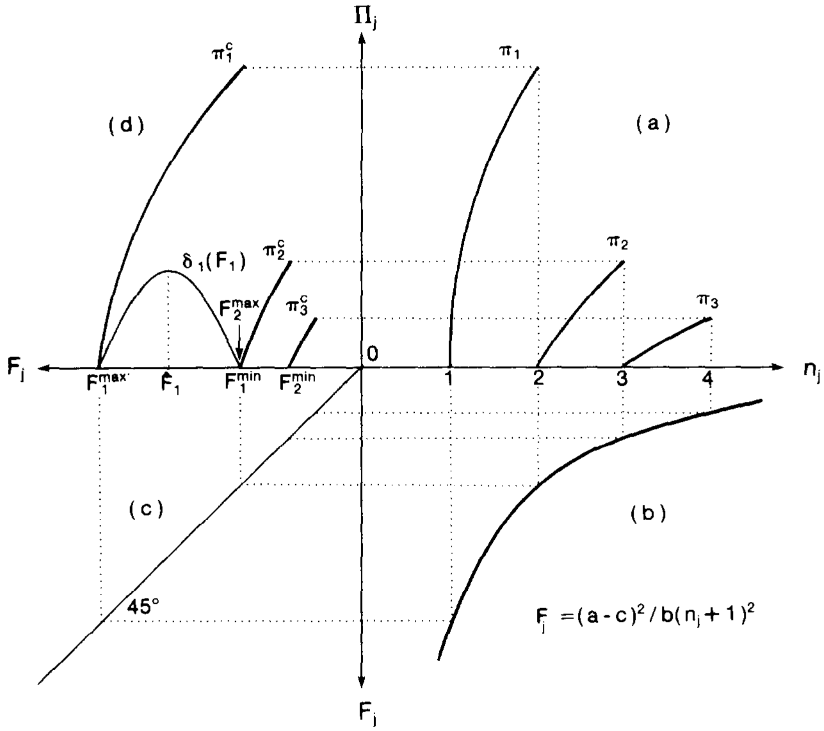

Fig. 1.

Let J be the set of industries with n_j^c active firms and let $\delta_j(F_j)$ be the distribution of fixed entry fees over $[F_j^{\min}, F_j^{\max})$; needless to say there is a one-to-one correspondence between $\delta_j(F_j)$ and a suitable distribution $\sigma_j(\mu_j)$ defined over (0, 1).

In order for a cross-industry regression of *average* profits onto any index of concentration based on the number of active firms to yield a *negative* coefficient, the distributions $\delta_j(F_j)$ must have the unusual property that the difference between $F_j^{\max}$ and $\hat{F}_j$ (the average fixed entry fee) is a decreasing convex function of n_j^c. Or, to put it differently, the average 'integer constraint effect' (i.e., the average value of μ_j) must be *larger* in *less* concentrated industries. In the absence of any economic reason why the *average size* of fixed entry fees should be systematically related to the *difference* between maximum and average fixed entry fees (i.e., $F_j^{\max} - \hat{F}_j$), we must conclude that the *size effect* can be expected, on average, to be stronger than the *integer constraint effect*, thereby yielding a *positive* correlation between concentration and profitability.

In conclusion, this note has shown that profitability and concentration are correlated even in a model with *both* free entry *and* no collusion. This is because with positive entry fees and non-fractional entry, free entry is compatible with positive super-normal profits being earned by active firms. These profits, however, are not correlated with concentration as such, but rather with the size of entry fees and the difference between the free-entry equilibrium of firms with and without fractional entry, i.e., μ_j. For a positive correlation to exist, it is sufficient that, on average, the latter is not inversely related to the size of fixed entry fees.

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